Solutions to JEE Main Home Practice Test - 1 | JEE - 2024

PHYSICS

SECTION-1

1.(B) $\lambda = \frac{h}{p}$ P = momentum $K.E. = \frac{P^2}{2m}$

$\lambda_P = \lambda_e$ $K.E. \propto \frac{1}{m}$

$K.E_P < K.E_e$

2.(A) $V = u + at$

$0 = u - gt$

$t = \frac{u}{g}$

Time taken to reach the ground

$t' = nt$

$t' = n \times \frac{u}{g}$

$S = ut + \frac{1}{2}at^2$

$H = -u\left(n \times \frac{u}{g}\right) + \frac{g}{2}\frac{n^2u^2}{g^2}$

$H = \frac{n^2}{2g}u^2 - \frac{nu^2}{g}$

$2gH = nu^2(n-2)$

3.(D) No-one is correct

4.(B) $P \propto V^2$

$P = KV^2$

$PV^{-2} = K$ (polytorpic process)

$PV^x =$ constant

Specific heat capacity $C = \frac{fR}{2} + \frac{R}{1-x}$

for diatomic gas $f = 5$, $C = \frac{5R}{2} + \frac{R}{1+2}$

$$C = \frac{5R}{2} + \frac{R}{3} = \frac{17R}{6}$$

5.(C) $u = -30\,cm$

$v = -75\,cm$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{-75}+\frac{1}{30}=\frac{1}{f}$$

$$\frac{1}{f}=\frac{30-75}{-75\times30}=\frac{45}{75\times30}$$

$$f=50\,cm$$

$$P=\frac{1}{f}$$

$$P=+2D$$

6.(D) $a\rightarrow$isobaric

$b\rightarrow$ isothermal

$c\rightarrow$adiabatic

$d\rightarrow$isochoric

7.(C) $B=\frac{\mu_0 I}{2R}\left(\frac{\theta}{2\pi}\right)$

$$B=\frac{\mu_0 I}{2R}\left(\frac{\theta}{2\pi}\right)-\frac{\mu_0 I}{4R}\frac{\theta}{2\pi}+\frac{\mu_0 I}{6R}\frac{\theta}{2\pi}$$

$$B=\frac{\mu_0 I}{2R}\frac{\theta}{2\pi}\left(1-\frac{1}{2}+\frac{1}{3}\right)$$

$$B=\frac{\mu_0 I}{2R}\frac{\theta}{2\pi}\left(\frac{5}{6}\right)$$

$$B=\frac{\mu_0}{4\pi}\left(\frac{5I\theta}{6R}\right)$$

8.(D) $L=Iw$

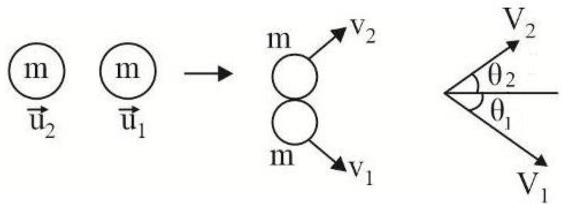
$$K.E.=\frac{1}{2}Iw^2$$

$$K.E.=\frac{1}{2}Lw$$

$$L=\frac{2K.E.}{w}$$

So, $L'=4L$

9.(D)



Momentum conservation in x direction

$$m_1u_1+m_2u_2=m_1v_1\cos\theta_1+m_2v_2\cos\theta_2$$

Momentum in y direction

$$0=m_1v_1\sin\theta_1-m_2v_2\sin\theta_2$$

$v_1 \sin\theta_1 = v_2 \sin\theta_2$

Elastic collision so K.E. is conserved

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

By solving we get $\theta_1 + \theta_2 = 90° = \frac{\pi}{2}$

10.(B) $B_{centre} = \frac{\mu_0 I}{2R}$

Flux linkage with r loop

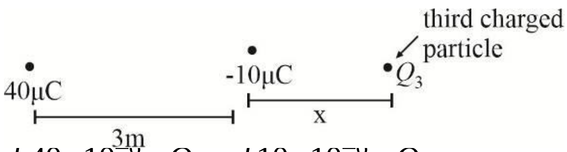
$\phi = B_{centre}\pi r^2$

$\phi = \frac{\mu_0 I}{2R}\pi r^2$

$\phi = M\ I$

$M = \frac{\mu_0}{2R}\pi r^2$

11.(B)



$$\frac{k40\times10^{-6}\times Q_3}{(3+x)^2} = \frac{k10\times10^{-6}\times Q_3}{x^2}$$

$x = 3m$

12.(A) MOI $I = mr^2 \quad [kg-m^2]$

$[M^1L^2T^0]$

Planck's constant $h = J-s$

$$= [M^1L^2T^{-2}][T^1] = [M^1L^2T^{-1}]$$

Pressure gradient $= \frac{dP}{dx}$

$$= \frac{[M^1L^{-1}T^{-2}]}{[L^1]} = [M^1L^{-2}T^{-2}]$$

Coefficient of elasticity $= N/m^2 = [M^1L^{-1}T^{-2}]$

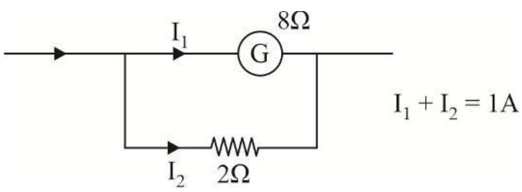
13.(C) $I_1 \times 8 = I_2 \times 2$

$4I_1 = I_2$

$I_1 + 4I_1 = 1$

$I_1 = 0.2A$

$I_2 = 0.8A$



14.(A) $KE = \frac{1}{2}mV^2$

For escape speed

$|P.E.| = |K.E.|_{required}$

$$K.E. = \frac{1}{2}m(nV_e)^2$$

$$KE = n^2\left(\frac{1}{2}mV_e^2\right)$$

Mechanical energy conservation

$$PE_i + KE_i = PE_f + KE_f$$

$$-\frac{1}{2}mV_e^2 + n^2\frac{1}{2}mV_e^2 = 0 + \frac{1}{2}mV_f^2$$

$$V_e^2(n^2 - 1) = V_f^2$$

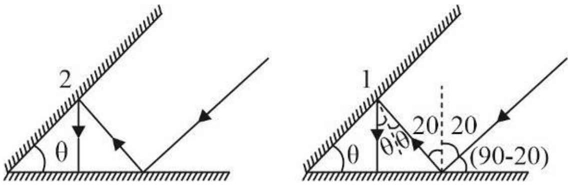
$$V_f = V_e\sqrt{(n^2 - 1)}$$

15.(B) Both lines are parallel so

$$\theta = 90 - 2\theta$$

$$3\theta = 90$$

$$\theta = 30°$$



16.(B) In first half upper diode in forward bias and lower diode in reverse bias
In second half lower diode in forward bias and upper diode in reverse bias

17.(D) $I = \frac{V}{Z}$

$$Z = \sqrt{R^2 + \left(Lw + \frac{1}{Cw}\right)^2}$$

$$f = 0,\ w = 0$$

$$z = \infty$$

$$I = 0$$

$$f = \infty,\ w = \infty$$

$$Z = \infty$$

$$I = 0$$

18.(A) Natural length is L

$$P = \gamma E$$

$$P = \gamma\frac{\Delta L}{L}$$

When m_1 hangs

$$\frac{M_1 g}{A} = \gamma\frac{L_1 - L}{L}$$

When mass m_1 & m_2 both hang

$$\frac{(M_1 + M_2)g}{A} = \gamma\frac{L_2 - L}{L}$$

$$\frac{M_1}{M_1 + M_2} = \frac{L_1 - L}{L_2 - L}$$

$$L_2M_1 - M_1L = L_1M_1 + L_1M_2 - LM_1 - LM_2$$

$$L = \frac{M_1(L_1 - L_2) + L_1M_2}{M_2}$$

$$L=\frac{M_1}{M_2}(L_1-L_2)+L_1$$

19.(A) $m_A.u_A = m_B.u_B - m_C u_C$

As $m_B = m_C$, $\frac{h}{\lambda_A} = m_B.u_B - m_B\frac{u_B}{2}$ AA

$$\frac{h}{\lambda_A}=\frac{1}{2}m_B.u_B=\frac{1}{2}\frac{h}{\lambda_B}$$

$$\lambda_B=\frac{\lambda_A}{2}$$

$$\lambda_C=\lambda_A$$

20.(A) Truth table

Input		Output
A	B	Y
1	1	1
1	0	1
0	1	1
0	0	0

OR gate

Boolean algebra

$$\overline{\overline{A}\cdot\overline{B}}=\overline{\overline{A}}+\overline{\overline{B}}=A+B$$

OR gate

SECTION - 2

1.(1) $\Delta p=-B\frac{\Delta V}{V}$

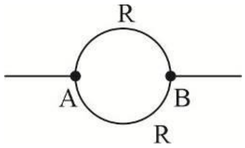
$$\rho gh=-B\left(\frac{0.2}{100}\right)$$

$$|\rho gh|=\left|B\frac{0.2}{100}\right|$$

$$10^3\times10\times200=B\times\frac{0.2}{100}$$

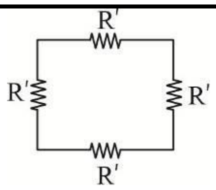
$$B=1\times10^9\ N/m^2$$

2.(6) Resistance $R\propto l$ (length)



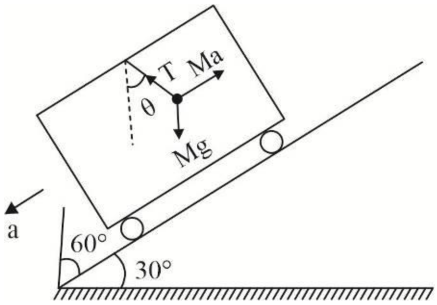
$R_{eq(AB)}=6\Omega$

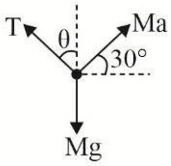
So, $R=12\Omega$



$R' = \frac{R}{2} = \frac{12}{2} = 6\Omega$

3.(60)





For equilibrium

$T\cos\theta = Mg - Ma\sin 30°$ …(i)

$T\sin\theta = Ma\cos 30°$ …(ii)

Equation (ii) / (i)

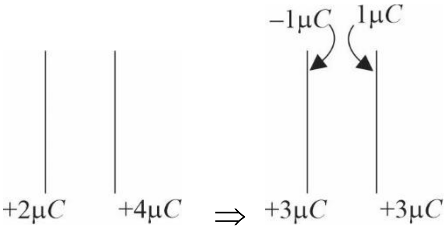
$\tan\theta = \frac{a\cos 30°}{g - a\sin 30°}$

$= \frac{\sqrt{3}}{2\left(1 - \frac{1}{2}\right)}$

$\tan\theta = \sqrt{3}$

$\theta = 60$

4.(1)



Pot. Diff $= \frac{Q}{C} = \frac{1\mu C}{1\mu F} = 1V$

5.(10) $f = (2n-1)\frac{V}{4L}$

$f = (2n-1)\frac{340}{4\times 1.7}$

$f = (2n-1)\times\frac{100}{2}$

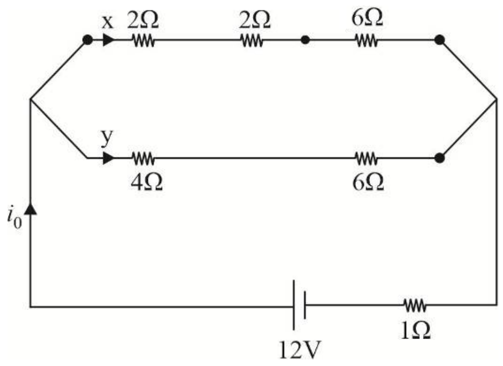
$f \le 1000$

$(2n-1)\times\frac{100}{2} \le 1000$

$n \le 10.5$

$n = 10$

6.(2)



On solving above circuit

$i_0 = 2A$

$x = 1A$

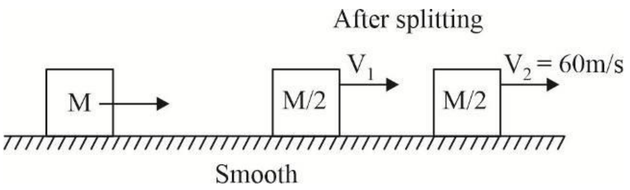
$y = 1A$

$V_{1\Omega} = 1 \times 2A = 2V$

7.(5) $mv = \frac{mv_1}{2} + \frac{mv_2}{2}$

$\Rightarrow \; v_1 = 2v - v_2$

$\Rightarrow \; v_1 = 20\, m/s$



Now,

$(K.E.)_{initial} = \frac{1}{2}mv^2 = 800m$

$(K.E.)_{final} = \frac{1}{2}\left(\frac{m}{2}\right)v_1^2 + \frac{1}{2}\left(\frac{m}{2}\right)v_2^2 = 1000\,m$

$\frac{K.E._{final}}{K.E._{initial}} = \frac{10}{8} = \frac{5}{4}$

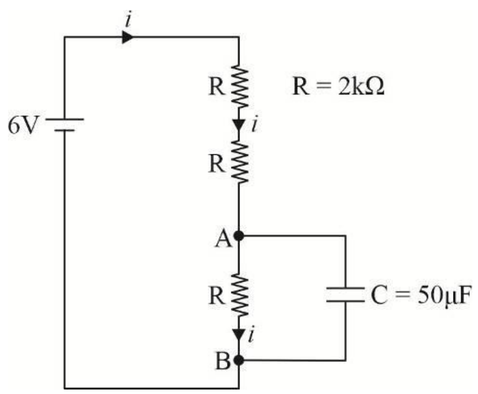
$\Rightarrow \; x = 5$

8.(100) Current through capacitor will be zero

$i = \frac{6}{3R}$

$\therefore \; V_{AB} = iR = 2V$

$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 50 \times 4\mu J = 100\mu J$



9.(6) $T = 2\pi\sqrt{\dfrac{m}{K}}$

$(P.E.)_{\max} = \dfrac{1}{2}KA^2$

When,

$\Rightarrow \; KE = \dfrac{PE}{3}$

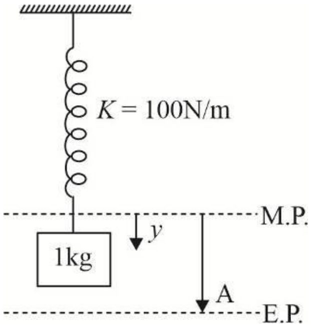
$\Rightarrow \; \dfrac{4}{3}PE = PE_{\max}$

This happens at y

$PE = \dfrac{3}{4}PE_{\max}$

$y = \dfrac{A\sqrt{3}}{2}, \; t = \dfrac{T}{6}$

$x = 6$



10.(500) $E = 50\sin\left(\omega t - \dfrac{\omega}{c}\cdot x\right)$

Here $E_0 = 50N/C$

Energy density $= \dfrac{1}{2}\varepsilon_0 E^2$

Energy stored in volume $(V) = \dfrac{1}{2}\varepsilon_0 E^2 \cdot V$

$5.5\times10^{-12} = \dfrac{1}{2}\times(8.8\times10^{-12})\times(50)^2\cdot V$

$V = \dfrac{5.5\times2}{2500\times8.8}$

$V = 5\times10^{-4}(m)^3$

$V = 5\times10^{2}(cm)^3$

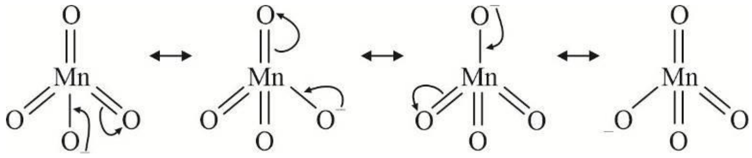
$V = 500(cm)^3$

CHEMISTRY

SECTION - 1

1.(C) In $K_3[Co(CN)_6] \rightarrow d^2sp^3$ hybridization (Inner orbital complex). So the coordination sphere is octahedral geometry and the ligands are which approaching the central metal atom along the coordinate axes, $d_{x^2-y^2}$ and d_{z^2} orbitals.

2.(D) MnO_4^- ion



B.O. of Mn – O $= 1 + \frac{\text{No.of } \pi \text{ bonds}}{\text{No.of } \sigma \text{ bonds}} = 1 + \frac{3}{4} = \frac{7}{4} = 1.75$

3.(D) $\Delta T_b = iK_b m$

The compound having highest van't Hoff factor (i) will have the highest boiling point.

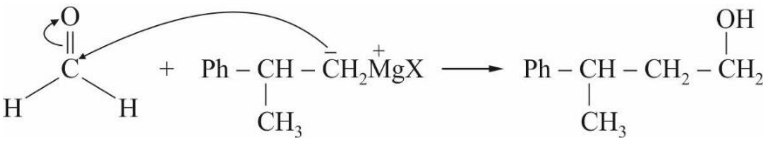
$Na_2SO_4 \longrightarrow 2Na^+ + SO_4^{2-}$; $i = 3$

$KCl \longrightarrow K^+ + Cl^-$; $i = 2$

urea $\longrightarrow i = 1$

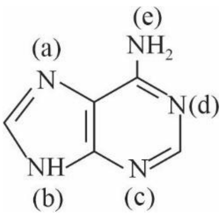
$K_3[Fe(CN)_6] \longrightarrow 3K^+ + [Fe(CN)_6]^{3-} : i = 4$

4.(A)



5.(A) Both (A) and (R) are correct and (R) is the correct explanation of (A).

6.(C)



Nitrogen atom marked (b) is attached to one H atom which can be removed by base.

7.(A) $C_2^{-2} \longrightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \left[\pi 2Px^2 = \pi 2Py^2\right] \sigma 2Pz^2$

Bond order $= \frac{10-4}{2} = 3(\text{diamagnetic})$

$N_2^{-2} \rightarrow \sigma 1s^2, \sigma * 1s^2, \sigma 2s^2, \sigma^* 2s^2, \left[\pi 2P_x^2 = \pi 2P_y^2\right] \sigma 2P_z^2 \left[\pi * 2P_x^1 = \pi * 2P_y^1\right]$

$B.O = \frac{10-6}{2} = 2(\text{Paramagnetic})$

$O_2^{-2} \rightarrow \sigma 1s^2, \sigma * 1s^2, \sigma 2s^2, \sigma * 2s^2, \sigma 2P_z^2 \left[\pi 2P_x^2 = \pi 2P_y^2\right]\left[\pi * 2P_x^2 = \pi * 2P_y^2\right]$

$$B.O = \frac{10-8}{2} = 1(\text{Diamagnetic})$$

$$O_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2P_z^2 \left[\pi 2P_x^2 = \pi 2P_y^2\right]\left[\pi^* 2P_x^1 = \pi^* 2P_y^1\right]$$

$$B.O = \frac{10-6}{2} = 2(\text{Paramagnetic})$$

$$B.O \propto \frac{1}{BL}$$

8.(D) $\frac{d[B]}{dt} = K_1[A] - K_2[B]$

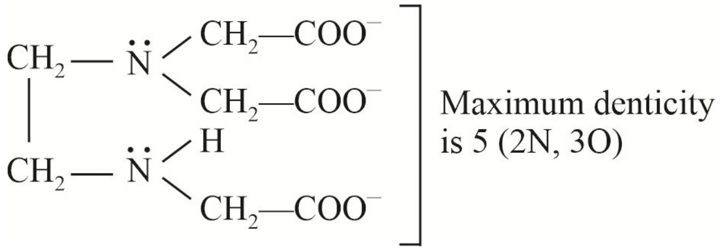
$$\frac{d[B]}{dt} = 0$$

$$\Rightarrow \quad [B] = \frac{K_1}{K_2}[A]$$

9.(A) $Ce : [Xe]4f^1 5d^1 6s^2$

Most common oxidation states ($+3 \rightarrow [Xe]4f^1$ & $+4 \rightarrow [Xe]$)

10.(C)



11.(B) By passing 0.1 Faraday electricity, 0.1 gm-equivalents of Ni^{+2} will be discharged.

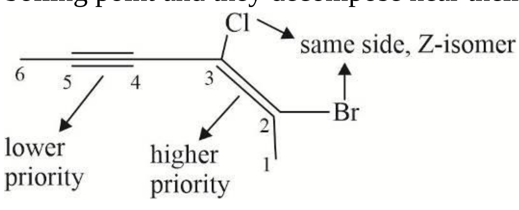
Number of gm-equivalent = (n – factor) × number of moles

$\Rightarrow$ 0.1 = 2 × number of moles

$\Rightarrow$ Number of moles $= \frac{0.1}{2} = 0.05$

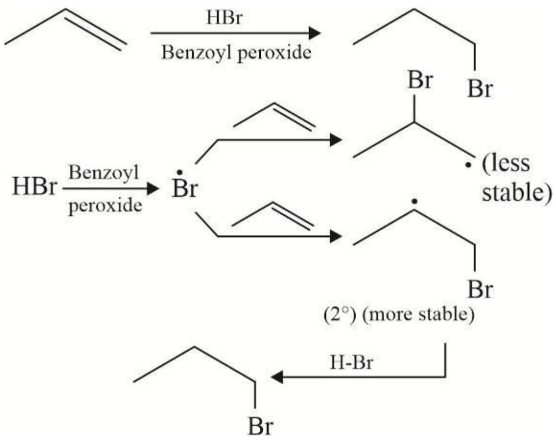
12.(A) Both are correct, and the reason behind vaccum distillation is that these compounds have very high boiling point and they decompose near their boiling points.

13.(C)

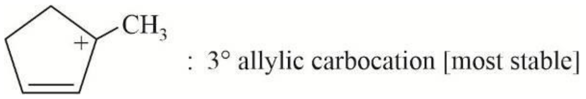


$\therefore$ (2Z)-2-Bromo-3-chlorohex-2-en-4-yne

14.(A)



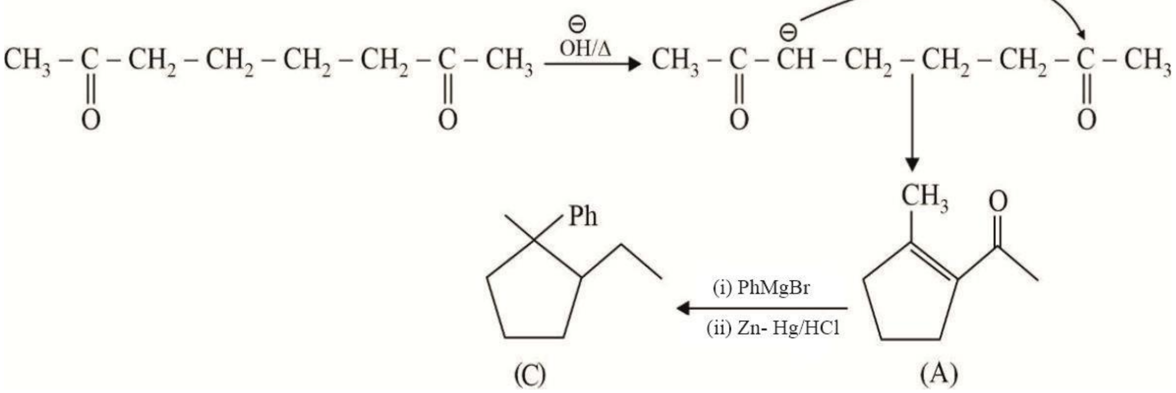
15.(B) Reactivity depend on extent of S_N1 mechanism

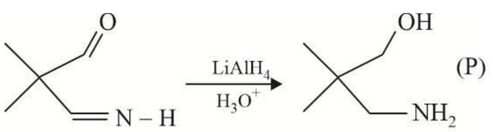


All the other compounds form less stable carbocations, hence, less reactive

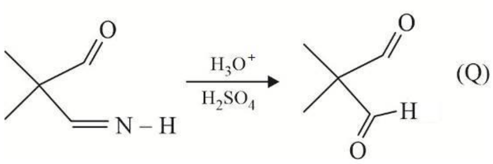
16.(C) Reaction will proceed via formation of most stable carbocation.
∴ major product is option (C).

17.(C)

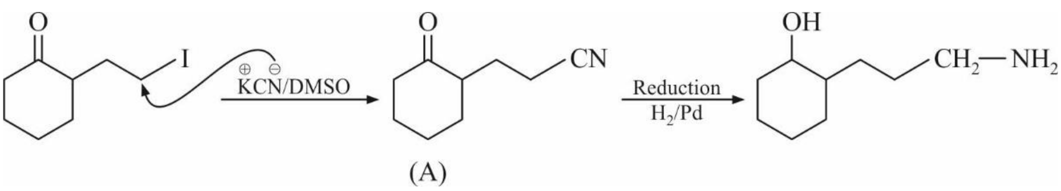




18.(C)



19.(B)



20.(A) In lactose, glycosidic linkage is between
(i) C4 of (ii) β-D-glucose & (iii) C1 of (iv) β-D galactose

SECTION - 2

1.(4) Washing soda : $Na_2CO_3 \cdot 10H_2O$

$M_0 = 286\,g/mol$

So, moles of washing soda $= \frac{57.2}{286} = 0.2\,mol$

Weight of solvent $= 1250 \times 1\,gm$

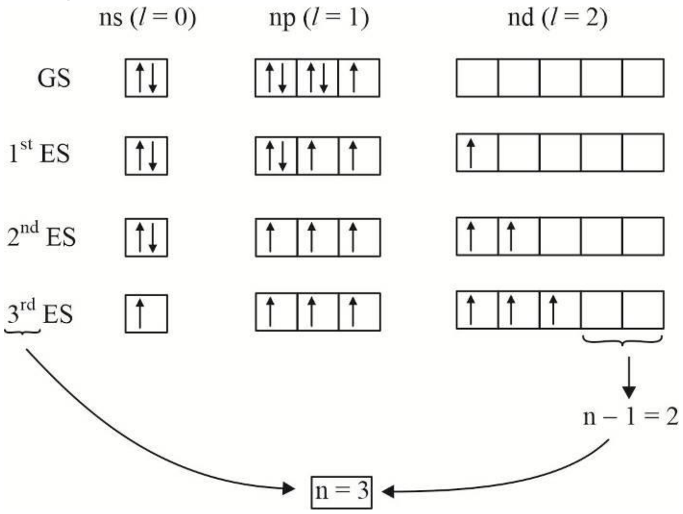
$= 1250\,gm = 1.25\,kg$

So, molality $= \frac{0.2}{1.25}\,mol/kg = 0.16 = 16 \times 10^{-2}\,mol/kg$

$y = 16$

$\therefore \sqrt{y} = 4$

2.(3) Halogens (except fluorine) :



3.(6) $k = Ae^{-E_a/RT}$

$\ln(k) = \left(\frac{-E_a}{R}\right)\frac{1}{T} + \ln A$

$\{y = mx + c\}$

$\frac{-E_a}{R} = -750$

$E_a = 6\,kJ/mol$

4.(1)

$$A_4B_3(s) \rightleftharpoons 4A^{3+}(aq) + 3B^{4-}(aq)$$

$t = 0$	-	-	-
$t = t_{eq}$	-	$4s_0$	$3s_0$

(s_o = solubility in Mol/L
s = Solubility in g/L)

$K_{sp} = [A^{3+}]^4[B^{4-}]^3 = (4s_0)^4(3s_0)^3$

$$= 4^4 \cdot 3^3 s_0^7 = 4^4 \cdot 3^3 \left(\frac{s}{M_0}\right)^7$$

So, $x = 4,\ y = 3,\ z = 7$

$$\therefore \quad \frac{x+y}{z} = \frac{4+3}{7} = 1$$

5.(5) Temperature coef. $= \dfrac{\Delta E}{\Delta T} = \dfrac{-0.01}{2} = -0.005\ \text{V/K}$

$\Rightarrow x = 0.005 \qquad \Rightarrow 1000\,x = 5$

6.(5) $kt = \ln \dfrac{[A]_0}{[A]_t}$

At 99.9% completion, $[A]_t = 0.1\%$ of $[A]_0 = \dfrac{[A]_0}{1000}$

$kt_{99.9\%} = \ln 1000$

$kt_{99.9\%} = 3\ln 10$

Also, $t_{1/2} = \dfrac{\ln 2}{k}$

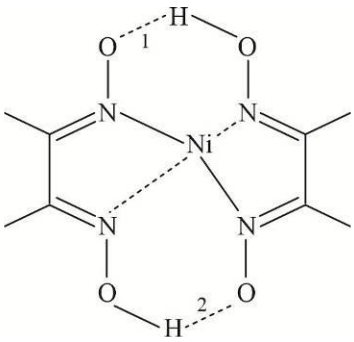
$t_{99.9\%} = 3 \times \dfrac{\ln 10}{\ln 2} \times t_{1/2}$

$t_{99.9\%} = \dfrac{3}{\log_{10} 2} t_{1/2}$

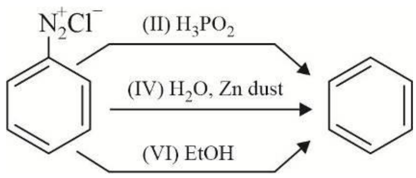
So, $n = \dfrac{3}{\log_{10} 2} = \dfrac{3}{0.3} = 10 \quad \therefore \ \dfrac{n}{2} = 5$

7.(1) Only chlorine forms halic (III) acid, i.e., chlorous acid (HOClO)

8.(2)



9.(3)



10.(2) The balanced equation is

$$BiO_3^- + 6H^+ + 2e^- \longrightarrow Bi^{+3} + 3H_2O$$

$x = 2$

MATHEMATICS

SECTION-1

1.(A) ${}_xR_y \rightarrow x-y+\sqrt{2}$ is an irrational number

Let R is a binary relation on real number x and y.

Clearly, R is reflexive relation

As ${}_xR_x$ iff $x-x+\sqrt{2}=\sqrt{2},$ which is an irrational number

Here R is not symmetric if we take $x=\sqrt{2}\ \&\ y=1$ then $x-y+\sqrt{2}$ is an irrational number but $y-x+\sqrt{2}=1,$ which is not irrational number

Now, R is transitive iff for all $(x,y)\in R\ \&\ (y,z)\in R$ implies $(x,z)\in R$

But here R is not transitive as we take $x=1,\ y=2\sqrt{2},\ z=\sqrt{2}$

Given, ${}_xR_y = x-y+\sqrt{2}$ is irrational ...(i)

And ${}_yR_z = y-z+\sqrt{2}$ is irrational ...(ii)

Add equation (i) and (ii), we get

$(x-y+\sqrt{2})+(y-z+\sqrt{2})=x-z+\sqrt{2}=1,$ which is not an irrational

2.(D) Let $e^x=t \quad \Rightarrow t^4+t^3-4t^2+t+1=0 \quad \Rightarrow \left(t^2+\frac{1}{t^2}\right)+\left(t+\frac{1}{t}\right)-4=0$

$\Rightarrow \left(t+\frac{1}{t}\right)^2+\left(t+\frac{1}{t}\right)-6=0 \quad \Rightarrow u^2+u-6=0 \quad \Rightarrow (u+3)(u-2)=0$

$t+\frac{1}{t}=2$; $t+\frac{1}{t}=-3$ (not possible)

$\Rightarrow t^2-2t+1=0 \quad \Rightarrow t=1 \quad \Rightarrow e^x=1 \quad \Rightarrow x=0$

3.(A) $a_r=e^{\frac{i2\pi r}{20}}$

$$a_r=e^{r\theta}=\begin{vmatrix} a_1 & a_3 & a_5 \\ a_7 & a_9 & a_{11} \\ a_{13} & a_{15} & a_{17}\end{vmatrix}=\begin{vmatrix} e^{\theta} & e^{3\theta} & e^{5\theta} \\ e^{7\theta} & e^{9\theta} & e^{11\theta} \\ e^{13\theta} & e^{15\theta} & e^{17\theta}\end{vmatrix}=e^{6\theta}\begin{vmatrix} e^{\theta} & e^{3\theta} & e^{5\theta} \\ e^{\theta} & e^{3\theta} & e^{5\theta} \\ e^{13\theta} & e^{15\theta} & e^{17\theta}\end{vmatrix}=0$$

4.(D) Consider the system of linear equations

$x+ky+3z=0$ ...(i)

$3x+ky-2z=0$ ...(ii)

$2x+4y-3z=0$ ...(iii)

For non-zero solutions to exist

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3\end{vmatrix}=0$$

Solving the above determinant equation, we get $-3k+8-k(-9+4)+3(12-2k)=0$

$\Rightarrow k=11$

Solving equations (i), (ii), and (iii) we get $2x=5z\ \&\ 3x=-(k+4)y$

$\Rightarrow\ 2x = 5z \,\&\, x = -5y$

Now, $\dfrac{xz}{y^2} = \dfrac{x \cdot \dfrac{2x}{5}}{\left(\dfrac{-x}{5}\right)^2} = 10$

5.(C) Let D be the common difference of the A.P.

Then, $a - 4b + 6c - 4d + e = a - 4(a + D) + 6(a + 2D) - 4(a + 3D) + (a + 4D) = 0$

6.(D) $b^2 = ac$

roots of $ax^2 + 2bx + c = 0$ are equal i.e. $-\dfrac{b}{a}$

$d\left(-\dfrac{b}{a}\right)^2 + 2e\left(-\dfrac{b}{a}\right) + f = 0$

$db^2 - 2bea + fa^2 = 0$

$dc - 2eb + fa = 0$

Divide by ac

$\dfrac{dc}{ac} - \dfrac{2eb}{b^2} + \dfrac{fa}{ac} = 0 \Rightarrow \dfrac{d}{a} - \dfrac{2eb}{b^2} + \dfrac{fa}{ac} = 0 \Rightarrow \dfrac{d}{a} - \dfrac{2e}{b} + \dfrac{f}{c} = 0$

$\dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in A.P.

7.(C) We have $\lim\limits_{x\to 0} \dfrac{\sin(2+x) - \sin(2-x)}{x} = \lim\limits_{x\to 0} \dfrac{2\cos\left(\dfrac{(2+x+2-x)}{2}\right)\sin\left(\dfrac{(2+x-2+x)}{2}\right)}{x}$

$= \lim\limits_{x\to 0} \dfrac{2\cos 2 \sin x}{x} = 2\cos 2 \lim\limits_{x\to 0} \dfrac{\sin x}{x} = 2\cos 2$

8.(B) Since $f(x)$ is continuous at $x = 2$

$\therefore\ f(2) = \lim\limits_{x\to 2^+} f(x) \qquad \Rightarrow\ 1 = \lim\limits_{x\to 2^+} (ax + b)$

$\therefore\ 1 = 2a + b \qquad \ldots$(i)

Again $f(x)$ is continuous at $x = 4$

$\therefore\ f(4) = \lim\limits_{x\to 4^-} f(x)$

$\Rightarrow\ 7 = \lim\limits_{x\to 4^-} (ax + b) = 4a + b \qquad \ldots$(ii)

Solving (i) and (ii), we get $a = 3,\ b = -5$

9.(B) $y' = \dfrac{1}{\sqrt{1 - \left(\dfrac{2x}{1+x^2}\right)^2}} \cdot \dfrac{2(1+x^2) - 4x^2}{(1+x^2)^2} = \dfrac{2(1-x^2)}{\sqrt{(1-x^2)^2(1+x^2)^2}} \Rightarrow\ y' = \begin{cases} \dfrac{2}{1+x^2} & for\ |x| < 1 \\ \dfrac{-2}{1+x^2} & for\ |x| > 1 \end{cases}$

Hence for $|x| = 1,$ the derivative does not exist

10.(D) Let $I = \int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx$

Substitute $u = \sqrt{x} \to du = \frac{1}{2\sqrt{x}} dx \quad \therefore \quad I = 2\int a^u du = \frac{2a^u}{\log a} + \lambda$

$= \frac{2a^{\sqrt{x}}}{\log a} + \lambda = 2a^{\sqrt{x}} \log_a e + \lambda$

11.(C) $\frac{dy}{dx} = e^{-2y}$

$\int e^{2y} dy = \int dx$

$\frac{e^{2y}}{2} = x + c$

$\frac{1}{2} = 5 + c$

$c = \frac{-9}{2}$

$x\,(\text{at } y = 3) = \frac{e^6 + 9}{2}$

12.(C) $\int_0^x \sqrt{1-(f'(t))^2}\, dt = \int_0^x f(t)\, dt$

Differentiating both sides using Leibnitz rule,

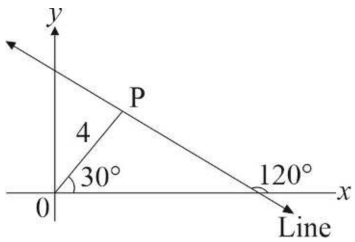
$f(x) = \sqrt{1-(f'(x))^2}, \quad 0 \le x \le 1 \qquad \Rightarrow \quad f(x) = \sin x, \quad 0 \le x \le 1$

And we know, $x > \sin x \forall x > 0 \qquad \Rightarrow \quad f(x) < x$

Hence option (C) is correct choice.

13.(A) Slope of $line = \tan(120°) = -\sqrt{3}$ and $P \equiv (4\cos 30°, 4\sin 30°) \equiv (2\sqrt{3}, 2)$

$\therefore$ Equation of line: $y + \sqrt{3}x = 8$.



14.(C) Given equation can be rewritten as

$(x-1)^2 + (y-3)^2 = \left(\frac{5x-12y+17}{13}\right)^2$

Here, focus is (1,3), directrix is $5x - 12y + 17 = 0$

$\therefore$ the distance of the focus from the directrix $= \left|\frac{5-36+17}{\sqrt{25+144}}\right| = \frac{14}{13}$ $\therefore$ Latusrectum $= 2 \times \frac{14}{13} = \frac{28}{13}$

15.(C) $be = 5\sqrt{3} \Rightarrow b^2e^2 = 75$

$b^2 - a^2 = 75$

$(b-a)(b+a) = 75$

$b + a = 15$

$b = 10, a = 5$

$LR = \frac{2a^2}{b} = \frac{2 \times 25}{10} = 5$

16.(C) Equations of straight line through the origin are $\frac{x-0}{l} = \frac{y-0}{m} = \frac{z-0}{n}$

where $l((b + c) + m(c + a) + n(a + b) = 0$ and $l(b - c) + m(c - a) + n(a - b) = 0$.

On solving, $\frac{l}{2(a^2 - bc)} = \frac{m}{2(b^2 - ca)} = \frac{n}{2(c^2 - ab)}$

Equations of the straight line is $\frac{x}{a^2 - bc} = \frac{y}{b^2 - ca} = \frac{z}{c^2 - ab}$

17.(D) $\sum_{k=1}^{n} (x-k)^2 = 0$

number of real root is zero.

18.(A) Let $\bar{R}$ be a vector in the plane of b and c

$\Rightarrow \bar{R} = (\hat{i} + 2\hat{j} - \hat{k}) + \mu(\hat{i} + \hat{j} - 2\hat{k})$.

Its projection on $\bar{a} = \frac{\bar{a}.\bar{R}}{|\bar{a}|} = \frac{1}{\sqrt{6}}$ $[2 + 2\mu - 2 - \mu - 1 - 2\mu] = \frac{-(1+\mu)}{\sqrt{6}}$

$\Rightarrow \frac{-(1+\mu)}{\sqrt{6}} = \pm\sqrt{\frac{2}{3}} \Rightarrow -(1+\mu) = \pm 2 \Rightarrow \mu = 1, -3$

$\Rightarrow R \equiv 2\hat{i} + 3\hat{j} - 3\hat{k}$ and $-2\hat{i} - \hat{j} + 5\hat{k}$.

Hence (A) is the correct answer.

19.(D) $\sin 10° \sin 30° \sin 50° \sin 70°$

$\cos 80° \left(\frac{1}{2}\right) \cos 40° \cos 20°$

$\frac{1}{2} \cos 20° \cos 40° \cos 80°$

$\frac{1}{2} \cdot \frac{\sin(2^3 \cdot 20°)}{2^3 \sin 20°}$

$\frac{1}{16} \frac{\sin 160°}{\sin 20°} = \frac{1}{16} \frac{\sin(180° - 20°)}{\sin 20°} = \frac{\sin 20°}{16 \sin 20°} = \frac{1}{16}$

20.(C) Let number of people in city = 100

$A \cap \bar{B} = 17$

30% of $\left(A \cap \bar{B}\right) = 5.1$

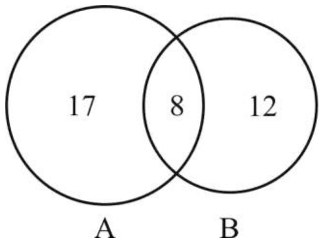
$\bar{A} \cap B = 12$

40% of $\left(\bar{A} \cap B\right) = 4.8$

$A \cap B = 8$

50% of $\left(A \cap B\right) = 4$

Total = 5.1 + 4.8 + 4 = 13.9



SECTION-2

1.(41) $T_{r+1} = {}^{10}C_r \cdot 2^{\frac{10-r}{2}} \cdot 3^{\frac{r}{5}}$

r = 0, 10

Sum $= {}^{10}C_0 \cdot 2^5 \cdot 3^0 + {}^{10}C_{10} \cdot 2^0 \cdot 3^2$ = 32 + 9 = 41.

2.(3) $z\bar{z} + (1-i)z + (1+i)\bar{z} - 7 = 0$

$\Rightarrow z\bar{z} + \bar{a}z + a\bar{z} - 7 = 0$ {where a = 1 + i}

$\Rightarrow$ radius of circle is given by

$r = \sqrt{|a|^2 - b} = \sqrt{2+7}$

$r = \sqrt{9}$

r = 3 units

3.(2) $x + \frac{a}{x^2} > 2 \, \forall x \in R^+$

Let $x^3 - 2x^2 + a > 0, \forall x > 0$

$f(x) = x^3 - 2x^2 + a$

$f'(x) = 3x^2 - 4x = x(3x-4)$

$f\left(\frac{4}{3}\right) > 0 \Rightarrow \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 + a > 0$

$\frac{64}{27} - \frac{32}{9} + a > 0$

$\frac{64-96}{27} + a > 0$

$a > \frac{32}{27}$

4.(0) $I = \int_0^{\pi} e^{\cos^2 x} \cos^3(2n+1)x \, dx = \int_0^{\pi} e^{\cos^2(\pi - x)} \cos^3(2n+1)(\pi - x) dx$

$$= \int_0^{\pi} e^{\cos^2 x} \cos^3 (2n\ \pi + \pi - (2n+1)\ x) dx = -\int_0^{\pi} e^{\cos^2 x} \cos^3 (2n+1)\ x\ dx = -\text{I}.$$

Hence $2\ I = 0 \Rightarrow I = 0$.

5.(18) $$f(x) = x^2 + \int_0^x e^{-t} f(x-t)\,dt = x^2 + \int_0^x e^{-(x-t)} f(t)\,dt = x^2 + \int_0^x e^{-x} \cdot e^t f(t)\,dt = x^2 + e^{-x}\int_0^x e^t f(t)\,dt$$

$$f'(x) = 2x + e^{-x} \cdot e^x f(x) - e^{-x}\int_0^x e^t f(t)\,dt = 2x + f(x) - f(x) + x^2$$

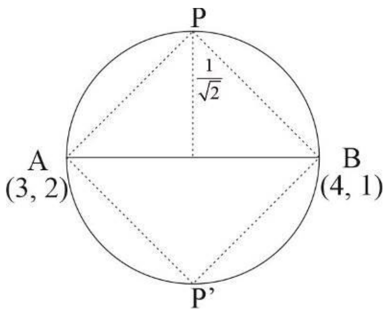
$$f(x) = x^2 + \frac{x^3}{3} + c$$

$$f(0) = 0$$

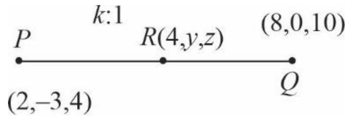
$$f(x) = x^2 + \frac{x^3}{3}$$

$$f(3) = 9 + 9 = 18$$

6.(2) $ar(\Delta APB) = \frac{1}{2} \times \sqrt{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$



7.(56)



$$\frac{8k+2}{k+1} = 4 \qquad \Rightarrow k = \frac{1}{2}$$

$\Rightarrow \qquad y = -2\ \ z = 6$

$\therefore \qquad R(4, -2, 6)$ Distance from origin $= 2\sqrt{14}$.

8.(132) Required ways $= \underline{\lfloor 12-6+1} \cdot \underline{\lfloor 6} = \underline{\lfloor 7}\,\underline{\lfloor 6}$

$\therefore$ Required probability $= \frac{\underline{\lfloor 7} \times \underline{\lfloor 6}}{\underline{\lfloor 12}} = \frac{1}{132}$

9.(576) The number of ways to fill the three even places by 4 consonants = ${}^4P_3$.
After filling the even places, remaining places can be filled in ${}^4P_4$ ways.
So, the required number of ways = ${}^4P_3 \times {}^4P_4 = 576$.
Hence, 576 is the correct answer.

10.(40) Co-efficient of variance = $\frac{\sigma}{\bar{x}} \times 100 \Rightarrow$ for first series $75 = \frac{15}{\bar{x}} \times 100 \Rightarrow \bar{x} = 20$

and for second series $90 = \frac{18}{\bar{x}} \times 100 \Rightarrow \bar{x} = 20$.

Thus, both the series have same mean i.e., 20.